Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 1

8 May 2023

Zone A afternoon | **Zone B** morning | **Zone C** afternoon

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

2223 – 7209

19 pages

Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 6]

The decathlon is a competition where athletes compete in ten events. Two of those events are long jump and high jump. In both events, a greater distance means a better ranking.

The table shows results for these two events at the World Championships.

	Event		Rank	
Athlete's Country	**Long Jump (m)**	**High Jump (m)**	**Long Jump Rank**	**High Jump Rank**
Germany	7.64	2.11	1	
France	7.52	2.08	2	
Estonia	7.49	1.84	3	
Canada	7.44	2.02	4	
Netherlands	7.33	2.05	5	
Ukraine	7.28	2.02	6	
Algeria	7.22	1.90	7	
Austria	7.11	1.87	8	
Grenada	6.98	1.99	9	
Japan	6.64	1.96	10	

The Spearman's rank correlation coefficient is used to determine if there is a linear correlation between an athlete's ranking in long jump and their ranking in high jump.

(a) Complete the table to show the athletes' rankings in high jump. [2]

(b) Find the value of the Spearman's rank correlation coefficient r_s. [2]

(This question continues on the following page)

(Question 1 continued)

The following guide is used by the coach to determine the strength of the correlation between the ranks for long jump and high jump.

$\lvert r_s \rvert$	**Strength**
0.000 to 0.199	Very weak
0.200 to 0.399	Weak
0.400 to 0.599	Moderate
0.600 to 0.799	Strong
0.800 to 1.000	Very strong

(c) State the strength of the correlation between the rankings as indicated by the table and interpret this in the context of the question. [2]

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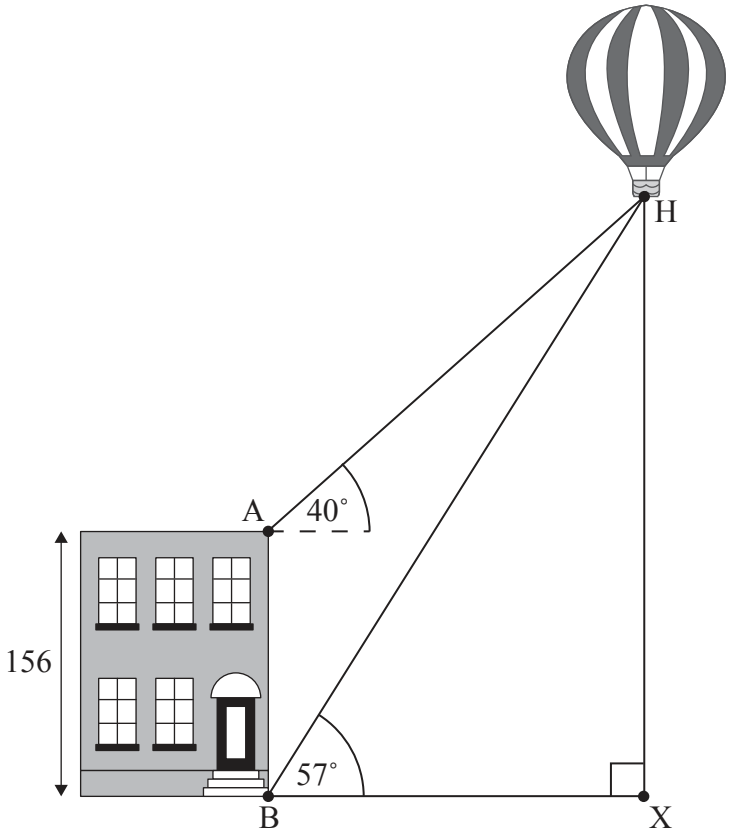
2. [Maximum mark: 6]

Point H on a hot-air balloon is sighted at the same time by two observers. One observer is at the top of a vertical building that is 156 metres tall. The other observer is at the base of the building.

The angle of elevation from point A (at the top of the building) to H is $40°$, and the angle of elevation from point B (at the base of the building) to H is $57°$. Point X is the ground directly below point H. This information is shown in the diagram.

diagram not to scale



(a) Find the size of angle $A\hat{H}B$. [2]

(b) Calculate the distance from point B to point H. [3]

The hot-air balloon remains at a constant height as it moves further away from the building.

(c) Describe, in words, the change in the angle of depression from point H to point B as the horizontal distance between the balloon and the building increases. [1]

(This question continues on the following page)

(Question 2 continued)

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3. [Maximum mark: 5]

On 1 January 2022, Mina deposited $\$1000$ into a bank account with an annual interest rate of 4%, compounded monthly. At the end of January, and the end of every month after that, she deposits $\$100$ into the same account.

(a) Calculate the amount of money in her account at the start of 2024. Give your answer to two decimal places. [3]

(b) Find how many complete months, counted from 1 January 2022, it will take for Mina to have more than $\$5000$ in her account. [2]

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4. [Maximum mark: 6]

Carys believes that, on a memory retention test, the mean score of bilingual people (μ_b) will be higher than the mean score of monolingual people (μ_m). Carys gave a memory retention test to a random sample of students in her class. The results are shown in the two tables.

	Scores									
Bilingual	100	94	100	90	100	94	98	98	98	98

	Scores							
Monolingual	97	92	88	98	88	94	100	100

Carys performs a one-tailed t-test at a 5% level of significance. It is assumed that the scores are normally distributed and the samples have equal variances.

(a) State the null and alternative hypotheses. [2]

(b) Calculate the p-value for this test. [2]

(c) State the conclusion of the test in the context of the question. Justify your answer. [2]

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5. [Maximum mark: 5]

Line L_1 is tangent to the graph of a function $f(x)$ at the point $\mathrm{P}(3, -1)$. Line L_2 is given by the equation $y = -\frac{1}{2}x - \frac{5}{2}$ and is perpendicular to L_1.

(a) Write down the gradient of L_1. [1]

(b) Find the equation of L_1 in the form $y = mx + c$. [2]

(c) Show that L_2 is not the line that is normal to $f(x)$ at point P. [2]

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6. [Maximum mark: 5]

When the brakes of a car are fully applied the car will continue to travel some distance before it completely stops. This stopping distance, d, in metres is directly proportional to the square of the speed of the car, v, in kilometres per hour ($\mathrm{km\,h^{-1}}$).

When a car is travelling at a speed of $50\,\mathrm{km\,h^{-1}}$ it will travel $12.3\,\mathrm{m}$ after the brakes are fully applied before it completely stops.

(a) Determine an equation for d in terms of v. [2]

The police can use this equation to estimate if cars are exceeding the speed limit.

A car is found to have travelled $33\,\mathrm{m}$, after fully applying its brakes, before it completely stopped.

(b) Use your equation from part (a) to estimate the speed at which this car was travelling before the brakes were applied. [2]

(c) After the brakes have been fully applied, identify one other variable besides speed that could affect stopping distance. [1]

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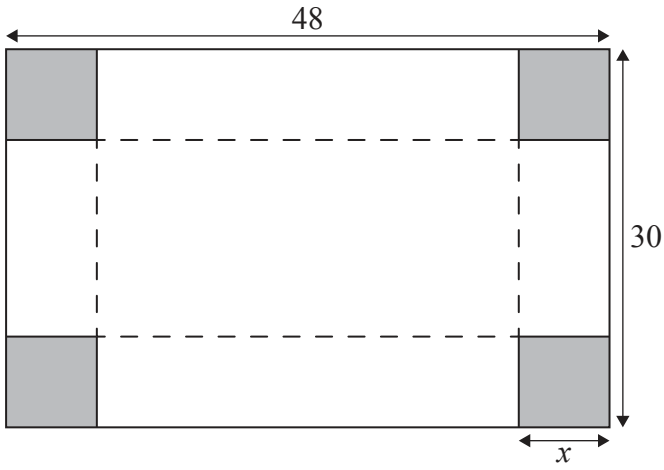
7. [Maximum mark: 6]

A rectangular box, with an open top, is to be constructed from a piece of cardboard that measures $48\,\text{cm}$ by $30\,\text{cm}$.

Squares of equal size will be cut from the corners of the cardboard, as indicated by the shading in the diagram. The sides will then be folded along the dotted lines to form the box.

diagram not to scale



The volume of the box, in cubic centimetres, can be modelled by the function $V(x) = (48 - 2x)(30 - 2x)(x)$, for $0 < x < k$, where x is the length of the sides of the squares removed in centimetres.

(a) Write down the maximum possible value of k in this context. [1]

(b) Find the value of x that maximizes the volume of the box. [2]

A second piece of $48\,\text{cm}$ by $30\,\text{cm}$ cardboard is damaged and a strip $2\,\text{cm}$ wide must be removed from all four sides. A box will then be constructed in a similar manner from the remaining cardboard.

(c) Calculate the maximum possible volume of the box made from the second piece of cardboard. [3]

(This question continues on the following page)

(Question 7 continued)

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8. [Maximum mark: 7]

"Password entropy" is a measure of the predictability of a computer password. The higher the entropy, the more difficult it is to guess the password.

The relationship between the password entropy, p, (measured in bits) and the number of guesses, G, required to decode the password is given by $0.301p = \log_{10} G$.

(a) Calculate the value of p for a password that takes 5000 guesses to decode. [2]

(b) Write down G as a function of p. [1]

(c) Find the number of guesses required to decode a password that has an entropy of 28 bits. Write your answer in the form $a \times 10^k$, where $1 \leq a < 10$, $k \in \mathbb{Z}$. [3]

There is a point on the graph of the function $G(p)$ with coordinates $(0, 1)$.

(d) Explain what these coordinate values mean in the context of computer passwords. [1]

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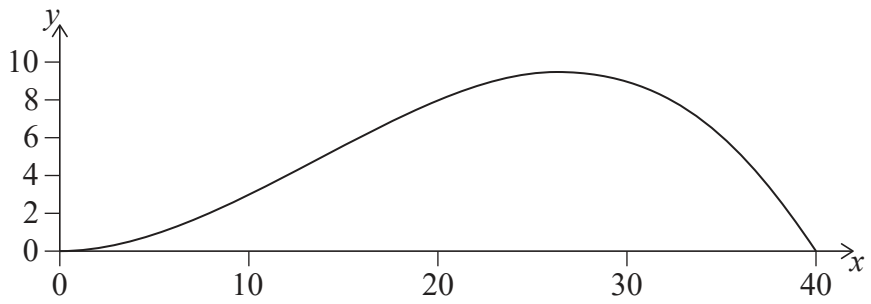
9. [Maximum mark: 8]

The cross section of a scale model of a hill is modelled by the following graph.



The heights of the model are measured at horizontal intervals and are given in the table.

Horizontal distance, x cm	0	10	20	30	40
Vertical distance, y cm	0	3	8	9	0

(a) Use the trapezoidal rule with $h = 10$ to find an approximation for the cross-sectional area of the model. [2]

It is given that the equation of the curve is $y = 0.04x^2 - 0.001x^3,\ 0 \le x \le 40$.

(b) (i) Write down an integral to find the exact cross-sectional area.

(ii) Calculate the value of the cross-sectional area to two decimal places. [4]

(c) Find the percentage error in the area found using the trapezoidal rule. [2]

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10. [Maximum mark: 7]

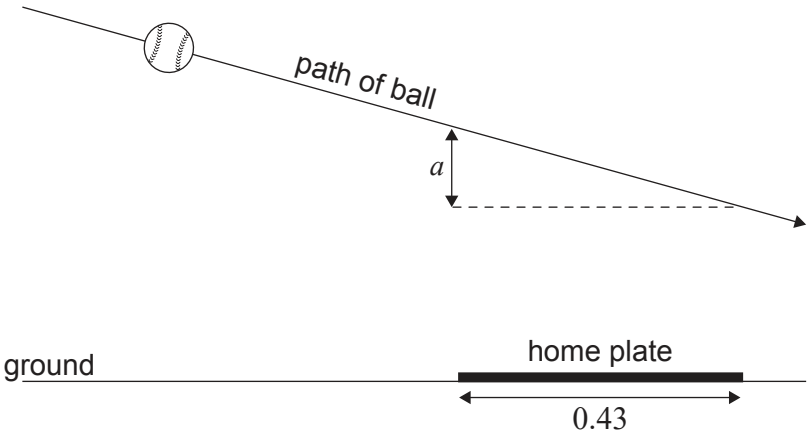
In a baseball game, Sakura is the batter standing beside home plate. The ball is thrown towards home plate along a path that can be modelled by the following function

$$y = -0.045x + 2.$$

In this model, x is the horizontal distance of the ball from the point the ball is thrown and y is the vertical height of the ball above the ground. Both measured in metres.

The outcome of the throw is called a strike if the height of the ball is between $0.53\,\text{m}$ and $1.24\,\text{m}$ at some point while it travels over home plate. The length of home plate is $0.43\,\text{m}$.

diagram not to scale



When the ball reaches the front of home plate, the height of the ball above the ground is $1.25\,\text{m}$. The height of the ball changes by a metres as the ball travels over the length of home plate.

(a) (i) Find the value of a.

(ii) Justify why this throw is a strike. [4]

On the next throw, Sakura hits the ball towards a wall that is 5 metres high. The horizontal distance of the wall from the point where the ball was hit is 96 metres. The path of the ball after it is hit can be modelled by the function $h(d)$.

$$h(d) = -0.01d^2 + 1.04d + 0.66, \text{ for } h,\ d > 0$$

In this model, h is the height of the ball above the ground and d is the horizontal distance of the ball from the point where it was hit. Both h and d are measured in metres.

(b) Determine whether the ball will go over the wall. Justify your answer. [3]

(This question continues on the following page)

(Question 10 continued)

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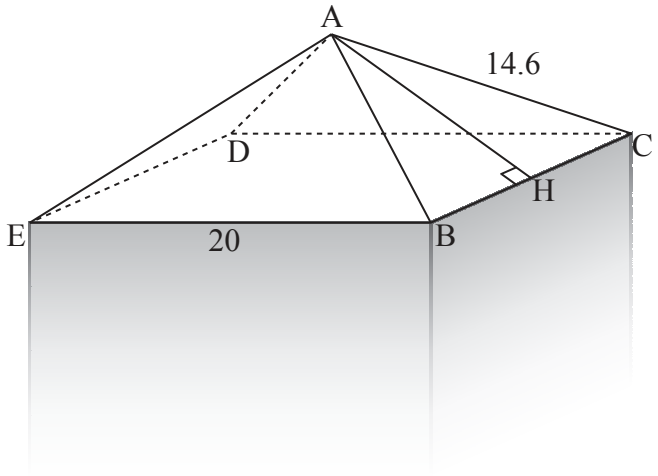
11. [Maximum mark: 7]

Vertical posts are to be placed around the outer edge of a children's park. Each post is formed from a cuboid with a right square-based pyramid on top.

The cuboid part of the post is machine-made such that its width, and hence the width of the pyramid, is exactly $20\,\text{cm}$. The length from the apex of the pyramid, A, to any corner of the base of the pyramid is $14.6\,\text{cm}$, **but** this is only accurate to the nearest tenth of a centimetre. The post is shown in the diagram.

diagram not to scale



(a) Write down the upper bound and lower bound for the possible lengths of edge AC. [2]

Point H is the midpoint of BC.

(b) Determine the upper bound and lower bound for AH, the slant height of the pyramid. [3]

For the post to be safe for children, the angle between the slant height and the base of the pyramid must be less than $22°$.

(c) Show that this post is safe for children. Justify your answer. [2]

(This question continues on the following page)

(Question 11 continued)

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12. [Maximum mark: 7]

On a specific day, the speed of cars as they pass a speed camera can be modelled by a normal distribution with a mean of $67.3\,\text{km h}^{-1}$.

A speed of $75.7\,\text{km h}^{-1}$ is two standard deviations from the mean.

(a) Find the standard deviation for the speed of the cars. [2]

Speeding tickets are issued to all drivers travelling at a speed greater than $72\,\text{km h}^{-1}$.

(b) Find the probability that a randomly selected driver who passes the speed camera receives a speeding ticket. [2]

It is found that 82% of cars on this road travel at speeds between $p\,\text{km h}^{-1}$ and $q\,\text{km h}^{-1}$, where $p < q$. This interval includes cars travelling at a speed of $74\,\text{km h}^{-1}$.

(c) Show that the region of the normal distribution between p and q is **not** symmetrical about the mean. [3]

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13. [Maximum mark: 5]

A boat travels $8\,\text{km}$ on a bearing of $315°$ and then a further $6\,\text{km}$ on a bearing of $045°$.
Find the bearing on which the boat should travel to return directly to the starting point.

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Answers written on this page
will not be marked.